JUNIOR CYCLE MATHS · FREE REVISION SHEET

Algebraic fractions & changing the subject

↗ Léim

Learning outcomes (2018 specification): AF.3a, AF.3c, AF.6. Play the game: **leim.ie/rearrange-race.html**

1. Adding and subtracting fractions

Just like number fractions: same bottom first, then add the tops. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$$\frac{x+3}{4} - \frac{x-2}{6}$$
$$= \frac{3(x+3) - 2(x-2)}{12} = \frac{3x + 9 - 2x + 4}{12} = \frac{x+13}{12}$$

Put each top in a **bracket**. The minus sign multiplies the **whole** bracket: −2 × −2 = +4. Never add the bottoms.

2. Long division

Divide, multiply, subtract, bring down. Same as 156 ÷ 12.

```
              x + 3
          ┌──────────
x + 2 │  x² + 5x + 6
      −(x² + 2x)
        ──────
            3x + 6
          −(3x + 6)
          ────────
                0
```

$x^2 \div x =$ **x** · $x(x + 2) = x^2 + 2x$ · subtract: $3x \cdot 3x \div x =$ **3** · $3(x + 2) = 3x + 6$ · remainder 0.
Check: $(x + 3)(x + 2) = x^2 + 5x + 6$ ✓

3. Changing the subject

Undo the steps in **reverse order**. y = 3x + 5:

x → **× 3** → **+ 5** → **y**

y → **− 5** → **÷ 3** → **x** **so x =** $\frac{y-5}{3}$

v = u + at → make t the subject

$v - u = at \rightarrow t = \frac{v-u}{a}$

y = (x + 4)/3 → make x the subject

A fraction? Multiply it away first: 3y = x + 4 → x = 3y − 4

4. Higher Level

$$\frac{3}{x+2} - \frac{1}{x-1}$$
$$= \frac{3(x-1) - 1(x+2)}{(x+2)(x-1)} = \frac{2x-5}{(x+2)(x-1)}$$

$A = \pi r^2 \rightarrow r$

$r^2 = A/\pi \rightarrow r = \sqrt{(A/\pi)}$ (square root last)

Subject twice: y = (x + 2)/(x − 3)

xy − 3y = x + 2 → xy − x = 3y + 2 → x(y − 1) = 3y + 2 → $x = \frac{3y+2}{y-1}$

$(x^3 - 8) \div (x - 2)$

Write $x^3 + 0x^2 + 0x - 8$ so nothing gets lost → $x^2 + 2x + 4$

Classic mistakes: adding the bottoms · not changing the sign of the second term after a minus · moving a number before multiplying away the fraction · dividing only one term · π outside the square root.

Exam tip: one step per line. After long division, multiply back to check. After rearranging, put in easy numbers to check both formulas agree.

PRACTICE · 32 QUESTIONS

Have a go

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A. Write as a single fraction (Q8: simplify)

1. $x/2 + x/5$ _____
2. $x/3 - x/4$ _____
3. $(x + 1)/2 + (x + 3)/4$ _____
4. $(x + 2)/3 + (x - 1)/2$ _____
5. $(2x + 1)/5 + (x - 3)/2$ _____
6. $(x + 4)/3 - (x - 2)/5$ _____
7. $(3x - 1)/4 - (x + 2)/6$ _____
8. $(6x + 9)/3$ _____

B. Divide

9. $(x^2 + 7x + 12) \div (x + 3)$ _____
10. $(x^2 + 2x - 15) \div (x + 5)$ _____
11. $(x^2 - 9x + 14) \div (x - 2)$ _____
12. $(2x^2 + 7x + 6) \div (x + 2)$ _____
13. $(3x^2 - 10x - 8) \div (x - 4)$ _____
14. $(6x^2 + x - 2) \div (2x - 1)$ _____

C. Change the subject

15. $y = x + 7$. Make x the subject. _____
16. $y = 4x - 3$. Make x the subject. _____
17. $y = (x - 5)/2$. Make x the subject. _____
18. $P = 4s$. Make s the subject. _____
19. $v = u + at$. Make u the subject. _____
20. $A = \frac{1}{2}bh$. Make b the subject. _____
21. $y = mx + c$. Make m the subject. _____
22. $F = 1.8C + 32$. Make C the subject, then find C when $F = 77$. _____

D. Higher Level

23. Higher Level: $3/(x + 1) + 2/(x - 4)$ _____
24. Higher Level: $5/(x - 2) - 3/(x + 3)$ _____
25. Higher Level: $(x^3 + 2x^2 - 5x - 6) \div (x + 1)$ _____
26. Higher Level: $(x^3 - 27) \div (x - 3)$ _____
27. Higher Level: $A = \pi r^2$. Make r the subject. _____
28. Higher Level: $y = (x + 4)/(x - 2)$. Make x the subject. _____

Questions 29–32 are on the next page.

EXAM-STYLE · THEN CHECK

Exam corner + answers

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E. Exam-style questions (show your work)

29. **One fraction, then solve.** (a) Write (x + 3)/2 + (x − 2)/3 as a single fraction. (b) Hence solve (x + 3)/2 + (x − 2)/3 = 5.

30. **Divide, then solve.** (a) Divide $x^2 + 3x - 28$ by x − 4. (b) Use your answer to solve $x^2 + 3x - 28 = 0$.

31. **The taxi.** A taxi fare is C = 3.5 + 2k euro for k km. (a) Make k the subject. (b) A fare is €15.50. How far was the trip?

32. **Higher Level.** $E = \frac{1}{2}mv^2$. (a) Make v the subject. (b) Find v when E = 400 and m = 8.

Answers

A	1. 7x/10 · 2. x/12 · 3. (3x + 5)/4 · 4. (5x + 1)/6 · 5. (9x − 13)/10 · 6. (2x + 26)/15 · 7. (7x − 7)/12 · 8. 2x + 3
B	9. x + 4 · 10. x − 3 · 11. x − 7 · 12. 2x + 3 · 13. 3x + 2 · 14. 3x + 2
C	15. x = y − 7 · 16. x = (y + 3)/4 · 17. x = 2y + 5 · 18. s = P/4 · 19. u = v − at · 20. b = 2A/h · 21. m = (y − c)/x · 22. C = (F − 32)/1.8; C = 25
D	23. (5x − 10)/((x + 1)(x − 4)) · 24. (2x + 21)/((x − 2)(x + 3)) · 25. $x^2 + x - 6$ · 26. $x^2 + 3x + 9$ · 27. r = √(A/π) · 28. x = (2y + 4)/(y − 1)
E	29. (a) (3(x + 3) + 2(x − 2))/6 = **(5x + 5)/6** (b) 5x + 5 = 30 → **x = 5** · 30. (a) **x + 7** (b) (x − 4)(x + 7) = 0 → **x = 4 or x = −7** · 31. (a) **k = (C − 3.5)/2** (b) (15.5 − 3.5)/2 = **6 km** · 32. (a) $2E = mv^2$ → **v = √(2E/m)** (b) √(800/8) = √100 = **10**

Got one wrong? Did you put each top in a bracket? Did the minus change BOTH signs? Did you multiply back to check your division? Did you undo the steps in reverse order? Then play **Rearrange Race** at leim.ie/rearrange-race.html: the "Go back over these" list at the end shows the rule for each mistake.

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