JUNIOR CYCLE MATHS · FREE REVISION SHEET

Factorising

↗ Léim

Learning outcome (2018 specification): AF.3d. Play the game: **leim.ie/factor-match.html**

Factorising is expanding **backwards**: put the brackets back. **Always check by multiplying out.**

1. Highest common factor (HCF)

Find the biggest number and the letters in **every** term (smallest power). Put it outside; divide each term by it.

$6x + 9$	**$3(2x + 3)$**
$4x^2 - 10x$	**$2x(2x - 5)$**
$6ab + 9ac - 3a$	**$3a(2b + 3c - 1)$** (keep the 1)
HCF of $12x^2y$, $18xy^2$	**$6xy$**

Fully factorised: nothing more can come out. $2(2x^2 - 5x)$ is **not** fully factorised.

2. Grouping (four terms)

$xy - 3x + 2y - 6$
$= x(y - 3) + 2(y - 3)$
$=$ **$(y - 3)(x + 2)$**

A minus in front of the second pair: $2ab + 8a - 3b - 12 = 2a(b + 4) - 3(b + 4) =$ **$(b + 4)(2a - 3)$**.

4. Difference of two squares

$a^2 - b^2 = (a - b)(a + b)$
$x^2 - 49 =$ **$(x - 7)(x + 7)$**
$2x^2 - 18 = 2(x^2 - 9) =$ **$2(x - 3)(x + 3)$**
HL: $9x^2 - 4y^2 =$ **$(3x - 2y)(3x + 2y)$**
$53^2 - 47^2 = 6 \times 100 =$ **600**

3. $x^2 + bx + c$

Find two numbers that **multiply** to c and **add** to b. $x^2 + 8x + 12$: pairs for 12:

1, 12 → 13	**2, 6 → 8 ✓**
3, 4 → 7	**−2, −6 → −8**

So $x^2 + 8x + 12 =$ **$(x + 2)(x + 6)$**.

Signs: c positive → both numbers have the sign of b. c negative → one +, one −.
$x^2 - 9x + 14 =$ **$(x - 2)(x - 7)$**
$x^2 + 2x - 15 =$ **$(x + 5)(x - 3)$**

Common factor first: $3x^2 + 15x + 18 = 3(x^2 + 5x + 6) =$ **$3(x + 2)(x + 3)$**

Higher Level: $ax^2 + bx + c$. Try brackets that give the first and last terms; check the middle. $2x^2 + 7x + 3 \rightarrow (2x + 1)(x + 3)$: middle $6x + x = 7x$ ✓. $(2x + 3)(x + 1)$ gives $5x$ ×.

Which method?

1. Common factor first, always. 2. Two terms, a minus, both squares → difference of two squares. 3. Three terms → $x^2 + bx + c$. 4. Four terms → grouping. Then **check by multiplying out**.

Exam tip: show the middle line in grouping, watch the signs ($x^2 - x - 12 = (x + 3)(x - 4)$), and check with $x = 1$: the question and your answer must give the same number.

PRACTICE · 34 QUESTIONS

Have a go

↗ Léim

A. Common factor: factorise

1. $8x + 12$ _____
2. $15a - 10$ _____
3. $x^2 + 7x$ _____
4. $6x^2 - 9x$ _____
5. $4ab + 6ac$ _____
6. $10xy - 15yz$ _____
7. $3p^2 + p$ _____
8. $6ab + 9ac - 3a$ _____

B. Grouping: factorise

9. $ax + ay + bx + by$ _____
10. $xy + 2x + 5y + 10$ _____
11. $ab - 3a + 4b - 12$ _____
12. $pq + 6p - 2q - 12$ _____
13. $3xy - 6x + y - 2$ _____
14. $ax - ay - bx + by$ _____

C. $x^2 + bx + c$: factorise

15. $x^2 + 7x + 12$ _____
16. $x^2 + 8x + 15$ _____
17. $x^2 - 9x + 20$ _____
18. $x^2 + 3x - 10$ _____
19. $x^2 - 2x - 24$ _____
20. $x^2 - 11x + 18$ _____
21. *HL* $2x^2 + 7x + 3$ _____
22. *HL* $3x^2 - 10x - 8$ _____

D. Squares and "fully"

23. $x^2 - 36$ _____
24. $y^2 - 1$ _____
25. *HL* $9x^2 - 25$ _____
26. *HL* $4a^2 - 49b^2$ _____
27. Factorise fully: $3x^2 - 12$ _____
28. Factorise fully: $2x^2 + 10x + 12$ _____
29. Use factorising to work out $65^2 - 35^2$ without a calculator. _____
30. Seán says $5x^2 + 10x = 5(x^2 + 2x)$ is fully factorised. Is he right? _____

HL = Higher Level. Questions 31–34 are on the next page.

EXAM-STYLE · THEN CHECK

Exam corner + answers

↗ Léim

E. Exam-style questions (show your work)

31. Factorise (a) $6x^2 - 15x$ (b) $xy + 4x - 3y - 12$ (c) $x^2 - 64$.

32. A rectangle has area $x^2 - 2x - 35$ cm². Its length is $x + 5$ cm. (a) Find its width. (b) Find the length, the width and the area when $x = 12$ and check.

33. (a) Factorise $x^2 + 6x + 9$. (b) What shape has area $x^2 + 6x + 9$? Give its side length.

34. (a) Factorise fully $4x^2 - 100$. (b) *Higher Level.* Factorise $2x^2 - 3x - 5$ and check with $x = 2$.

Answers

A	1. $4(2x + 3)$ · 2. $5(3a - 2)$ · 3. $x(x + 7)$ · 4. $3x(2x - 3)$ · 5. $2a(2b + 3c)$ · 6. $5y(2x - 3z)$ · 7. $p(3p + 1)$ · 8. $3a(2b + 3c - 1)$
B	9. $(x + y)(a + b)$ · 10. $(y + 2)(x + 5)$ · 11. $(b - 3)(a + 4)$ · 12. $(q + 6)(p - 2)$ · 13. $(y - 2)(3x + 1)$ · 14. $(x - y)(a - b)$
C	15. $(x + 3)(x + 4)$ · 16. $(x + 3)(x + 5)$ · 17. $(x - 4)(x - 5)$ · 18. $(x + 5)(x - 2)$ · 19. $(x + 4)(x - 6)$ · 20. $(x - 2)(x - 9)$ · 21. $(2x + 1)(x + 3)$ · 22. $(3x + 2)(x - 4)$
D	23. $(x - 6)(x + 6)$ · 24. $(y - 1)(y + 1)$ · 25. $(3x - 5)(3x + 5)$ · 26. $(2a - 7b)(2a + 7b)$ · 27. $3(x - 2)(x + 2)$ · 28. $2(x + 2)(x + 3)$ · 29. $(65 - 35)(65 + 35) = 30 \times 100 = 3{,}000$ · 30. No: $x^2 + 2x$ still has x in both terms. $5x(x + 2)$
E	31. (a) **$3x(2x - 5)$** (b) $x(y + 4) - 3(y + 4) =$ **$(y + 4)(x - 3)$** (c) **$(x - 8)(x + 8)$** · 32. (a) $(x + 5)(x - 7)$, so width **$x - 7$** cm (b) 17 cm × 5 cm = 85 cm²; $144 - 24 - 35 =$ **85** ✓ · 33. (a) **$(x + 3)(x + 3)$** $= (x + 3)^2$ (b) a **square** with sides $x + 3$ · 34. (a) $4(x^2 - 25) =$ **$4(x - 5)(x + 5)$** (b) **$(2x - 5)(x + 1)$**; $x = 2$: $8 - 6 - 5 = -3$ and $(-1)(3) = -3$ ✓

Got one wrong? Multiply your answer out: does it give the question back? Did you take out the **highest** common factor? Did you lose a 1? Are the signs right (multiply to c, add to b)? Then play **Factor Match** at leim.ie/factor-match.html: the "Go back over these" list at the end shows the rule for each mistake.

All questions written by Léim. Léim is independent and not connected with the State Examinations Commission or the NCCA.