JUNIOR CYCLE MATHS · FREE REVISION SHEET

Linear patterns & sequences

↗ Léim

Learning outcomes (2018 specification): N.4, AF.1a, AF.1b, U.4, U.11. Play the game: **leim.ie/pattern-builder.html**

1. The words

A **sequence** is a list of numbers in order: 5, 8, 11, 14, ... Each number is a **term**: $T_1 = 5$, $T_2 = 8$. $\mathbf{T_n}$ is the nth term.

In a **linear** pattern you add (or take away) the **same amount** every time: the **common difference**. 5, 8, 11, ... goes up by **3**; 20, 16, 12, ... by **−4**.

2. Finding T_n (the general term)

1. Find the common difference d: the rule starts **dn**.
2. Compare dn with T_1, then add or take away.
3. **Check** with T_2.

5, 8, 11, ...: d = 3, so 3n. 3(1) = 3, but $T_1 = 5$: add 2. $\mathbf{T_n = 3n + 2}$. Check: 3(2) + 2 = 8 ✓

30, 26, 22, ...: d = −4. −4(1) = −4, $T_1 = 30$: add 34. $\mathbf{T_n = 34 - 4n}$.

Not "n + 3": that only says how to get from one term to the next. Test it: n = 1 gives 4, not 5.

3. Picture patterns



Stage 1 Stage 2 Stage 3

Each new stage adds **2** matchsticks, and stage 1 has 3 = 2 + **1**, so stage n has **2n + 1**. Stage 10 has 21, **not** 10 × 3 = 30.

4. Far-away terms, working backwards

50th term of 5, 8, 11, ...: 3(50) + 2 = **152** (or 5 + **49** jumps of 3).

Which term is 65? (65 − 5) ÷ 3 = 20 jumps after T_1, so it's $\mathbf{T_{21}}$. Check: 3(21) + 2 = 65 ✓

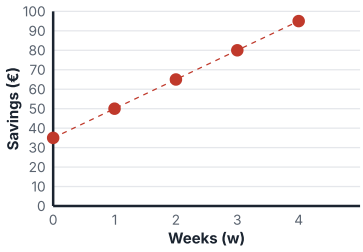
Is 100 a term? (100 − 5) ÷ 3 = 31.66..., not whole, so **no**.

5. Tables and graphs

n	1	2	3	4
$\mathbf{T_n}$	5	8	11	14

Plot the points (n, T_n): a linear pattern makes a **straight line**.

6. Real life



€35 saved, plus €15 a week. **Start value** = 35 (week 0, where the line meets the axis). **Rate of change** = 15 per week. **S = 15w + 35**.

Exam tip: fill in tables carefully (later parts use them), check every rule with two terms, and show your working for "which stage?" questions: (100 − 1) ÷ 3 = 33.

PRACTICE · 34 QUESTIONS

Have a go

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A. Continue the pattern

1. Write the next two terms: 4, 9, 14, 19, … _____
2. Write the next two terms: 30, 27, 24, 21, … _____
3. Write the next two terms: −7, −3, 1, 5, … _____
4. Write the next two terms: 11, 20, 29, 38, … _____
5. Fill in the gap: 6, __, 20, 27 _____
6. Fill in the gap: 45, 39, __, 27 _____
7. Fill in the gap: __, 2, 9, 16 _____
8. What is the common difference of 52, 45, 38, 31, …? _____

B. Find the rule

9. Find T_n for 5, 8, 11, 14, … _____
10. Find T_n for 9, 13, 17, 21, … _____
11. Find T_n for 1, 7, 13, 19, … _____
12. Find T_n for 2, 7, 12, 17, … _____
13. Find T_n for 20, 18, 16, 14, … _____
14. Find T_n for −3, 0, 3, 6, … _____
15. Find the rule for this table: n = 1, 2, 3, 4 → 12, 19, 26, 33 _____
16. Write the first four terms of $T_n = 6n - 4$ _____

C. Far-away terms and working backwards

17. Find the 20th term of 3, 7, 11, 15, … _____
18. Find the 100th term of 3, 7, 11, 15, … _____
19. Which term of 3, 7, 11, 15, … is 83? _____
20. Is 100 a term of 3, 7, 11, 15, …? Explain. _____
21. Find the 50th term of 90, 88, 86, 84, … _____
22. Which term of 7, 13, 19, 25, … is 157? _____
23. Is 95 a term of 5, 14, 23, 32, …? If so, which? _____
24. $T_n = 8n - 3$. Which term is 117? _____

D. Pictures and real life

25. The matchstick triangles at the top of page 1: how many matchsticks in stage 4? _____
26. …and in stage 20? _____
27. …write the rule for stage n. _____
28. Ella has €40 and saves €12 a week. Write a formula for her savings S after w weeks. _____
29. …how much will she have after 8 weeks? _____
30. …after how many weeks will she have €220? _____

Questions 31–34 are on the next page.

EXAM-STYLE · THEN CHECK

Exam corner + answers

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E. Exam-style questions (show your work)

31. Tiles: stage 1 has 5 tiles, stage 2 has 8, stage 3 has 11. (a) Make a table for stages 1 to 5. (b) Find a formula for the number of tiles in stage n. (c) How many tiles are in stage 30? (d) Which stage uses 125 tiles?

32. The sequence 50, 46, 42, 38, ... (a) Find T_n. (b) Find T_{10}. (c) Which is the first term that is less than zero? Explain.

33. Hannah's plant is 8 cm tall and grows 3 cm a week. (a) Write a formula for its height h after w weeks. (b) Find the height after 10 weeks. (c) After how many weeks will it be 50 cm? (d) On a graph of h against w, what are the start value and the rate of change?

34. Data plans: Plan A costs €10 plus €2 per GB. Plan B costs €4 per GB. (a) Make a table of the cost of each plan for 0 to 6 GB. (b) For how many GB do the plans cost the same? (c) Which plan is cheaper for 8 GB?

Answers

A	1. 24, 29 · 2. 18, 15 · 3. 9, 13 · 4. 47, 56 · 5. 13 · 6. 33 · 7. −5 · 8. −7
B	9. 3n + 2 · 10. 4n + 5 · 11. 6n − 5 · 12. 5n − 3 · 13. 22 − 2n · 14. 3n − 6 · 15. 7n + 5 · 16. 2, 8, 14, 20
C	17. 79 · 18. 399 · 19. 21 · 20. No: (100 − 3) ÷ 4 = 24.25, not a whole number · 21. −8 · 22. 26 · 23. Yes: term 11 · 24. 15
D	25. 9 · 26. 41 · 27. 2n + 1 · 28. S = 12w + 40 · 29. €136 · 30. 15
E	31. (a) 5, 8, 11, 14, 17 (b) **3n + 2** (c) 3(30) + 2 = **92** (d) (125 − 2) ÷ 3 = **stage 41** · 32. (a) **54 − 4n** (b) 54 − 40 = **14** (c) T_{13} = 2 and T_{14} = −2, so $\mathbf{T_{14} = -2}$ · 33. (a) **h = 3w + 8** (b) **38 cm** (c) (50 − 8) ÷ 3 = **14 weeks** (d) start value **8 cm**, rate of change **3 cm per week** · 34. (a) A: 10, 12, 14, 16, 18, 20, 22; B: 0, 4, 8, 12, 16, 20, 24 (b) **5 GB** (both €20) (c) A: €26, B: €32, so **Plan A**

Got one wrong? Check: did you count the jumps (the 20th term is 19 jumps after the 1st)? Did you put the common difference in front of the n? Did you check your rule with the second term? Did you work backwards before dividing? Then play **Pattern Builder** at leim.ie/pattern-builder.html: the "Go back over these" list at the end shows the rule for each mistake.

All questions written by Léim. Léim is independent and not connected with the State Examinations Commission or the NCCA.