JUNIOR CYCLE MATHS · FREE REVISION SHEET

Quadratic equations

↗ Léim

Learning outcomes (2018 specification): AF.4, AF.5, U.6. Play the game: **leim.ie/root-rush.html**

1. What is a root?

A **root** (or solution) is a value of x that makes the equation true. A quadratic (an x^2 equation) usually has **two** roots. Check a root by putting it in: x = 2 in $x^2 - 5x + 6 = 0$ gives $4 - 10 + 6 = 0$ ✓

2. Solve in 3 steps

1. Make it = 0
$x^2 + 2x = 15 \rightarrow x^2 + 2x - 15 = 0$
2. Factorise
$(x + 5)(x - 3) = 0$
3. Each bracket = 0
$x + 5 = 0 \rightarrow x = -5 \cdot x - 3 = 0 \rightarrow x = 3$

The roots **multiply to the last number** ($-5 \times 3 = -15$) and **add to the middle number with its sign changed** ($-5 + 3 = -2$, middle term $+2x$).

3. Two short cuts

$x^2 - 6x = 0 \rightarrow x(x - 6) = 0 \rightarrow x = 0$ or $x = 6$
$x^2 - 49 = 0 \rightarrow x^2 = 49 \rightarrow x = 7$ or $x = -7$

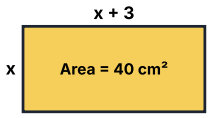
Never divide by x: you lose x = 0.

4. Roots → equation

Roots 4 and $-2 \rightarrow (x - 4)(x + 2) = 0 \rightarrow x^2 - 2x - 8 = 0$

The sign in the bracket is the **opposite** of the root.

5. Word problems



$x(x + 3) = 40 \rightarrow x^2 + 3x - 40 = 0 \rightarrow (x + 8)(x - 5) = 0$. x = −8 or x = 5, but a length can't be negative: **x = 5 cm**. Always say which root you reject and why.

6. Higher Level

$2x^2$ …: $2x^2 + x - 6 = 0 \rightarrow (2x - 3)(x + 2) = 0 \rightarrow x = 3/2$ or $x = -2$.

The formula: $x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$. $x^2 - 4x - 3 = 0$: $x = (4 \pm \sqrt{28}) \div 2 = 4.65$ or -0.65.

Find k: x = 3 is a root of $x^2 + kx - 21 = 0 \rightarrow 9 + 3k - 21 = 0 \rightarrow k = 4$.

Classic mistakes: not making it = 0 first · a term moved without changing sign · root 3 from (x + 3) · only one root for $x^2 = 49$ · keeping a negative length.

Exam tip: write the factors, then both roots, then check one by putting it back in. In a word problem, answer the question that was asked, with units.

PRACTICE · 32 QUESTIONS

Have a go

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A. Roots

1. Is $x = 3$ a root of $x^2 - 5x + 6 = 0$? _____
2. Is $x = -2$ a root of $x^2 + x - 6 = 0$? _____
3. Solve $(x - 4)(x + 1) = 0$. _____
4. Solve $x(x - 9) = 0$. _____
5. Solve $(x + 6)(x + 2) = 0$. _____
6. Make it = 0: $x^2 + 4x = 12$. _____

B. Solve each equation

7. $x^2 + 5x + 6 = 0$
8. $x^2 - 8x + 15 = 0$
9. $x^2 + 2x - 8 = 0$
10. $x^2 - x - 12 = 0$
11. $x^2 - 7x = 0$
12. $x^2 + 4x = 0$
13. $x^2 - 64 = 0$
14. $x^2 = 81$
15. $x^2 + 3x = 10$
16. $x^2 = 6x - 8$

C. Roots → equation

17. Write an equation with roots 3 and 5. _____
18. Write an equation with roots −2 and 7. _____
19. Write an equation with roots 0 and −4. _____
20. Write an equation with roots 6 and −6. _____
21. $x = 2$ is one root of $x^2 + 3x - 10 = 0$. Find the other root. _____
22. Higher Level: $x = 4$ is a root of $x^2 + kx - 12 = 0$. Find k. _____

D. Word problems and Higher Level

23. A rectangle is x cm wide and $(x + 4)$ cm long. Its area is 60 cm². Find x. _____
24. The square of a positive number, plus 3 times the number, is 40. Find the number. _____
25. Two whole numbers in a row multiply to 156. Find them. _____
26. A ball's height is $h = 25t - 5t^2$ metres after t seconds. When does it land? _____
27. Higher Level: solve $3x^2 - 5x - 2 = 0$. _____
28. Higher Level: solve $x^2 + 6x + 4 = 0$, giving your answers to 2 decimal places. _____

Questions 29–32 are on the next page.

EXAM-STYLE · THEN CHECK

Exam corner + answers

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E. Exam-style questions (show your work)

29. **Solve and check.** Solve $x^2 - 2x - 24 = 0$. Check both roots.

30. **Roots.** (a) Write an equation with roots −3 and 5. (b) Solve $x^2 - 9 = 0$ and $x^2 - 9x = 0$.

31. **A patio.** A rectangular patio is x m wide and (x + 2) m long. Its area is 35 m². (a) Show that $x^2 + 2x - 35 = 0$. (b) Find the length and width of the patio.

32. **Higher Level.** (a) Solve $2x^2 - 7x + 3 = 0$. (b) Solve $2x^2 + 3x - 4 = 0$, giving your answers to 2 decimal places.

Answers

A	1. yes: 9 − 15 + 6 = 0 ✓ · 2. no: 4 − 2 − 6 = −4 · 3. x = −1 or x = 4 · 4. x = 0 or x = 9 · 5. x = −6 or x = −2 · 6. $x^2 + 4x - 12 = 0$
B	7. x = −3 or x = −2 · 8. x = 3 or x = 5 · 9. x = −4 or x = 2 · 10. x = −3 or x = 4 · 11. x = 0 or x = 7 · 12. x = −4 or x = 0 · 13. x = −8 or x = 8 · 14. x = −9 or x = 9 · 15. x = −5 or x = 2 · 16. x = 2 or x = 4
C	17. $x^2 - 8x + 15 = 0$ · 18. $x^2 - 5x - 14 = 0$ · 19. $x^2 + 4x = 0$ · 20. $x^2 - 36 = 0$ · 21. x = −5 · 22. k = −1
D	23. $x^2 + 4x - 60 = 0$ → x = 6 cm (x = −10 rejected) · 24. $x^2 + 3x - 40 = 0$ → 5 · 25. 12 and 13 · 26. t = 5 s · 27. x = −1/3 or x = 2 · 28. x = −5.24 or x = −0.76
E	29. (x − 6)(x + 4) = 0: **x = 6 or x = −4**. Check: 36 − 12 − 24 = 0 ✓, 16 + 8 − 24 = 0 ✓ · 30. (a) (x + 3)(x − 5) = 0, **$x^2 - 2x - 15 = 0$** (b) **x = 3 or x = −3**; x(x − 9) = 0, **x = 0 or x = 9** · 31. (a) x(x + 2) = 35 (b) (x + 7)(x − 5) = 0, x = 5 (not −7): **5 m by 7 m** · 32. (a) (2x − 1)(x − 3) = 0, **x = 1/2 or x = 3** (b) x = (−3 ± √41) ÷ 4: **−2.35 or x = 0.85**

Got one wrong? Check: did you make it = 0 first? Do your roots multiply to the last number? Did you give both roots? Then play **Root Rush** at leim.ie/root-rush.html: the "Go back over these" list at the end shows the rule for each mistake.

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