JUNIOR CYCLE MATHS · FREE REVISION SHEET

Quadratic & exponential patterns

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Learning outcomes (2018 specification): AF.1a, AF.1b, AF.1c, N.4. Play the game: **leim.ie/growth-spotter.html**

1. Three kinds of pattern

Linear: 5, 8, 11, 14
Same jump every time (+3). Graph: a straight line.

Quadratic: 3, 6, 11, 18
Jumps 3, 5, 7 change by the same amount: same **2nd difference** (+2). Graph: a curve.

Exponential: 3, 6, 12, 24
Same **multiplier**: × 2 (doubling) or × 3 (tripling). Graph: flat, then shoots up.

2. Differences

```
1    4    9    16    25
   3    5    7     9      ← 1st differences
     2    2    2          ← 2nd differences
```

Next term of a quadratic: 3, 6, 11, 18, ? Jumps 3, 5, 7, so the next jump is 9: 18 + 9 = **27**.

3. Growing tile patterns



Stage 1 Stage 2 Stage 3

Stage n = an n × n square + 2 yellow tiles: **$n^2 + 2$**.
Stage 10 = 10 × 10 + 2 = **102**. (Not 10 × stage 1.)

4. Doubling and tripling

1, 2, 4, 8, … × 2 each time. 1, 3, 9, 27, … × 3 each time.

Start 50, doubles every hour. After 3 hours?
$50 \times 2 \times 2 \times 2 = 50 \times 2^3 =$ **400**

Rules: 2, 4, 8, 16 → **$T_n = 2^n$**. 3, 9, 27, 81 → **$T_n = 3^n$**.
Don't mix up 2n (2, 4, 6, 8), n^2 (1, 4, 9, 16) and 2^n (2, 4, 8, 16).

5. Higher Level: $T_n = an^2 + bn + c$

4, 11, 22, 37: 1st differences 7, 11, 15; 2nd differences 4, 4. **a = half the 2nd difference = 2.**

n	1	2	3	4
T_n	4	11	22	37
$2n^2$	2	8	18	32
$T_n - 2n^2$	2	3	4	5

The last row is n + 1, so **$T_n = 2n^2 + n + 1$**.
Check n = 2: 8 + 2 + 1 = 11 ✓
Which term is 211? $2n^2 + n + 1 = 211$ → n = 10.

Classic mistakes: adding the same jump again in a quadratic · stage 10 = 10 × stage 1 · doubling as + 2 instead of × 2 · a = the 2nd difference (it's half) · 2n, n^2 and 2^n mixed up.

Exam tip: always give a reason: write the differences (or the × number). Check any rule with two terms from the table.

PRACTICE · 32 QUESTIONS

Have a go

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A. Linear, quadratic or exponential? Give a reason.

1. 4, 9, 14, 19, 24, … _____
2. 2, 6, 18, 54, 162, … _____
3. 1, 5, 11, 19, 29, … _____
4. 7, 14, 28, 56, 112, … _____
5. 10, 13, 18, 25, 34, … _____
6. 40, 35, 30, 25, 20, … _____

B. Write the next two terms

7. 4, 7, 12, 19, … _____
8. 1, 2, 4, 8, … _____
9. 2, 8, 18, 32, … _____
10. 5, 15, 45, 135, … _____
11. 3, 8, 15, 24, … _____
12. 6, 11, 18, 27, … _____

C. Differences and terms

13. Write the 1st and 2nd differences of 2, 7, 16, 29, 46.

    _____

14. Find the missing term: 3, 7, ?, 21, 31 (quadratic)

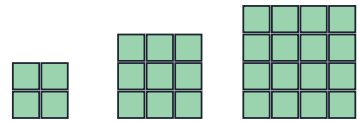
    _____

15. Find the missing term: 2, ?, 12, 20, 30 (quadratic)

    _____

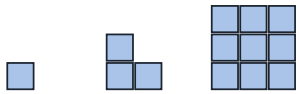
16. $T_n = n^2 + 7$. Find T_5. _____
17. $T_n = 2^n$. Find T_8. _____

D. Growing pictures and doubling



Stage 1 Stage 2 Stage 3

18. How many tiles in stage 4? _____
19. Linear, quadratic or exponential? Why? _____
20. How many tiles in stage 10? _____



Stage 1 Stage 2 Stage 3

21. How many tiles in stage 4? _____
22. How many tiles in stage 6? _____
23. Bacteria double every hour. Start: 30. How many after 5 hours? _____

E. Higher Level: find $T_n = an^2 + bn + c$

24. 3, 6, 11, 18, … _____
25. 4, 10, 18, 28, … _____
26. 5, 11, 21, 35, … _____
27. 0, 5, 14, 27, … _____
28. $T_n = n^2 + 3n$. Which term is 130? _____

Questions 29–32 are on the next page.

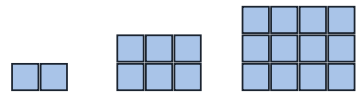
EXAM-STYLE · THEN CHECK

Exam corner + answers

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F. Exam-style questions (show your work)

29. **Tiles.** The pattern has 2, 6, 12 tiles in stages 1, 2, 3. (a) How many tiles in stage 4? (b) Linear, quadratic or exponential? Give a reason. (c) How many tiles in stage 10?



Stage 1 Stage 2 Stage 3

30. **Two plans.** Plan A pays €50 a week (50, 100, 150, ...). Plan B pays €1 in week 1 and doubles every week (1, 2, 4, ...). In which week is Plan B first bigger than Plan A?

31. **Folding paper.** Each fold doubles the layers: 1 fold → 2 layers. (a) Layers after 6 folds? (b) Fewest folds for more than 500 layers?

32. **Higher Level.** 6, 15, 28, 45, ... is quadratic. (a) Find $T_n = an^2 + bn + c$. (b) Which term is 231?

Answers

A	1. Linear · 2. Exponential · 3. Quadratic · 4. Exponential · 5. Quadratic · 6. Linear (same jump / same 2nd difference / same × number)
B	7. 28, 39 · 8. 16, 32 · 9. 50, 72 · 10. 405, 1215 · 11. 35, 48 · 12. 38, 51
C	13. 1st: 5, 9, 13, 17; 2nd: 4, 4, 4 · 14. 13 · 15. 6 · 16. 32 · 17. 256
D	18. 25 · 19. quadratic (jumps 5, 7, 9; 2nd difference 2) · 20. 121 (11 × 11) · 21. 27 · 22. 243 · 23. 960
E	24. $T_n = n^2 + 2$ · 25. $T_n = n^2 + 3n$ · 26. $T_n = 2n^2 + 3$ · 27. $T_n = 2n^2 - n - 1$ · 28. n = 10
F	29. (a) jumps 4, 6 → 8: **20** (b) 2nd differences 2, 2: **quadratic** (c) n(n + 1) = 10 × 11 = **110** · 30. Week 9: 256 < 450. Week 10: 512 > 500. **Week 10** · 31. (a) 2^6 = **64** (b) 2^8 = 256, 2^9 = 512: **9 folds** · 32. (a) 2nd difference 4, a = 2; $T_n - 2n^2$ = 4, 7, 10, 13 = 3n + 1: **$T_n = 2n^2 + 3n + 1$** (b) $2n^2 + 3n + 1 = 231$ → (2n + 23)(n − 10) = 0 → **n = 10**

Got one wrong? Did you write out the differences? Did you multiply (not add) for doubling? Did you use the rule for far-away stages? Then play **Growth Spotter** at leim.ie/growth-spotter.html: the "Go back over these" list at the end shows the rule for each mistake.

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