JUNIOR CYCLE MATHS · FREE REVISION SHEET

Probability 2: relative frequency & counting

↗ Léim

Learning outcomes (2018 specification): SP.1b, SP.2c. Play the game: **leim.ie/coin-dice-lab.html**

1. Relative frequency

Relative frequency = times it happened ÷ number of trials

A drawing pin is dropped 50 times and lands point up 18 times: 18 ÷ 50 = **0.36** (= 36% = 9/25). Divide by the number of **trials**, not by "the rest".

2. Estimating a probability

When outcomes are not equally likely (a pin, a bent coin, a penalty), do an experiment: the relative frequency is your **estimate** of the probability. The more trials, the better the estimate.

Tosses	10	100	1,000
Heads	7	46	503
Rel. freq.	0.7	0.46	0.503

To combine results, **add the counts first**: 12 of 30 and 18 of 50 → 30 of 80 = 0.375.

3. Expected frequency

Expected frequency = probability × number of trials

Fair die, 300 rolls: expect 1/6 × 300 = **50** sixes (about 50, not exactly). Pin from section 1, 1,000 drops: 0.36 × 1,000 = **360**.

4. Fair or biased?

Compare the results with what you'd expect. A die rolled 600 times: expect about 100 of each.

Score	1	2	3	4	5	6
Die A	96	104	99	107	93	101
Die B	78	81	74	83	79	205

Die A: all close to 100, so **fair**. Die B: a 6 came up about twice as often as expected, so probably **biased**. Only a few trials? You can't tell.

5. The fundamental principle of counting

m ways, then n ways → m × n ways together

3 starters, 4 mains, 2 desserts: 3 × 4 × 2 = **24** meals. A lock with 3 dials (0–9): 10 × 10 × 10 = **1,000** codes.

6. When you can't repeat

The choices go down: 4 friends in a row: 4 × 3 × 2 × 1 = **24**. Gold, silver and bronze from 8 runners: 8 × 7 × 6 = **336**. A 3-dial code set at random: P(7 1 3) = **1/1,000**.

Exam tip: show the division ("18 ÷ 50 = 0.36"), write the answer the way the question asks (decimal, % or fraction), give reasons with numbers ("a 6 came up 205 times but we'd expect about 100"), and **multiply, don't add**, when counting.

PRACTICE · 34 QUESTIONS

Have a go

↗ Léim

A. Relative frequency (give decimals unless asked otherwise)

1. A drawing pin is dropped 50 times and lands point up 18 times. Find the relative frequency of point up. _____
2. Aoife takes 40 penalties and scores 34. Find the relative frequency of scoring. _____
3. Cian flips a bottle 80 times and it lands standing up 12 times. Find the relative frequency as a percentage. _____
4. Of 250 seeds planted, 215 grew. Find the relative frequency of a seed growing. _____
5. A coin is tossed 20 times and lands heads 9 times. Find the relative frequency of heads as a fraction. _____
6. In a survey of 200 cars, 46 were red. Estimate the probability that the next car is red. _____
7. A die is rolled 120 times and a 6 comes up 26 times. Relative frequency of a 6 as a fraction? _____
8. A cup is thrown 25 times and lands on its side 17 times. Find the relative frequency of landing on its side. _____

B. Results tables

Score	1	2	3	4	5	6
Frequency	31	36	33	30	34	36

9. The table shows 200 rolls of a die (above). Find the relative frequency of a 2. _____
10. Same die: find the relative frequency of an even number. _____
11. A second die is rolled 120 times: 1 → 17, 2 → 21, 3 → ?, 4 → 19, 5 → 24, 6 → 20. How many 3s? _____
12. Ciara: pin dropped 30 times, 12 point up. Seán: 50 times, 18 point up. Combined relative frequency? _____
13. Who has the better estimate in question 12, Ciara or Seán? Why? _____
14. A die rolled 600 times gives 98, 101, 104, 97, 99, 101. Does it look fair? Why? _____

C. Expected frequency

15. A fair die is rolled 300 times. How many times would you expect a 5? _____
16. A fair coin is tossed 250 times. How many heads would you expect? _____
17. A spinner has 8 equal sectors, 3 blue. Expected blues in 400 spins? _____
18. 2 red and 3 green counters; one is picked and put back 150 times. Expected reds? _____
19. Using question 1, how many times would you expect the pin to land point up in 1,000 drops? _____
20. P(the bus is late) = 0.15. On how many of the next 60 school days would you expect it to be late? _____
21. P(red) = 1/4. How many spins are needed to expect red 30 times? _____
22. A bag of 20 counters: picked and put back 200 times, red 70 times. Estimate the red counters. _____

D. The fundamental principle of counting

23. A café has 4 starters, 5 mains and 3 desserts. How many lunch deals (one of each)? _____
24. Saoirse has 5 tops and 3 pairs of jeans. How many outfits (top + jeans)? _____
25. A bike lock has 4 dials, each 0 to 9. How many codes? _____
26. A code is one letter A to Z then one digit 0 to 9. How many codes? _____
27. In how many ways can 5 friends stand in a row? _____
28. 10 runners: in how many ways can gold, silver and bronze be won? _____
29. 3-digit codes from the digits 1 to 6, no digit used twice. How many? _____
30. A hoodie: 6 colours, 4 sizes, with or without a name. How many types? _____

Questions 31–34 are on the next page.

EXAM-STYLE · THEN CHECK

Exam corner + answers

↗ Léim

E. Exam-style questions (show your work)

31. A paper spinner has 3 colours. It is spun 200 times: red 46, blue 92, green 62. (a) Find the relative frequency of each colour. (b) Estimate how many times it would land on blue in 1,500 spins. (c) Niamh says the spinner must be fair because every colour came up. Is she right? Explain.

32. Darragh and Róisín test the same drawing pin. Darragh: 20 drops, 14 point up. Róisín: 180 drops, 99 point up. (a) Find each relative frequency. (b) Whose estimate is better? Why? (c) Put the results together to estimate the probability. (d) Estimate how many times it lands point up in 600 drops.

33. A pizza shop offers 3 bases, 6 toppings and 2 sizes. A pizza is one base, one topping and one size. (a) How many different pizzas? (b) A pizza is picked at random. Find P(it has the chilli topping). (c) The shop adds more toppings and now has 54 different pizzas. How many toppings now?

34. A phone PIN has 4 digits, each 0 to 9. (a) How many PINs are possible? (b) How many start with 0? (c) How many have four different digits? (d) A PIN is picked at random. Find P(all four digits are different) as a decimal.

Answers

A	1. 0.36 · 2. 0.85 · 3. 15% · 4. 0.86 · 5. 9/20 · 6. 0.23 · 7. 13/60 · 8. 0.68
B	9. 0.18 · 10. 0.51 · 11. 19 · 12. 0.375 · 13. Seán (more trials) · 14. fair: all close to 100
C	15. 50 · 16. 125 · 17. 150 · 18. 60 · 19. 360 · 20. 9 · 21. 120 · 22. 7
D	23. 60 · 24. 15 · 25. 10,000 · 26. 260 · 27. 120 · 28. 720 · 29. 120 · 30. 48
E	31. (a) red **0.23**, blue **0.46**, green **0.31** (b) 0.46 × 1,500 = **690** (c) **No**: if fair, each would be about 200 ÷ 3 ≈ 67; blue came up 92 times and red only 46, so it looks biased. · 32. (a) **0.7** and **0.55** (b) **Róisín's**: more trials (c) 113 of 200 = **0.565** (d) 0.565 × 600 = **339** · 33. (a) 3 × 6 × 2 = **36** (b) 3 × 1 × 2 = 6 of 36: **1/6** (c) 54 ÷ (3 × 2) = **9** · 34. (a) **10,000** (b) 1 × 10 × 10 × 10 = **1,000** (c) 10 × 9 × 8 × 7 = **5,040** (d) 5,040 ÷ 10,000 = **0.504**

Got one wrong? Check: did you divide by the number of trials? Did you multiply by the number of trials for "expect"? Did you multiply (not add) the choices when counting? When you can't repeat, do the choices go down? Then play **Coin & Dice Lab** at leim.ie/coin-dice-lab.html: the "Go back over these" list at the end shows the rule for each mistake.

All questions written by Léim. Léim is independent and not connected with the State Examinations Commission or the NCCA.