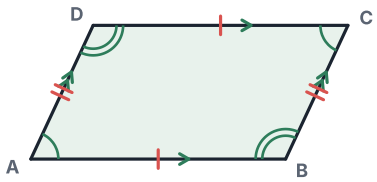
JUNIOR CYCLE MATHS · FREE REVISION SHEET

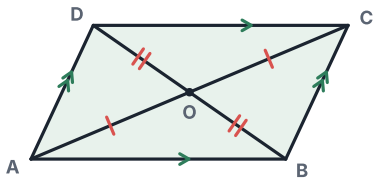
Theorems, congruent & similar triangles

↗ Léim

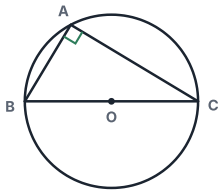
Learning outcomes (2018 specification): GT.3b, GT.3c, GT.3d, GT.3e, U.11, U.12. Play the game: **leim.ie/proof-builder.html**



Opposite sides equal, opposite angles equal (Theorem 9)



The diagonals bisect each other (Theorem 10)



The angle in a semicircle is 90° (Corollary 3)

1. Words of proof

Axiom: accepted without proof. **Theorem:** proved. **Corollary:** follows quickly from a theorem. **Converse:** swap "if" and "then" (not always true!). **Implies (⇒):** if ... then .... **Proof:** steps, each with a reason.

2. Parallelograms

Opposite sides equal, opposite angles equal, neighbouring angles add to 180° (co-interior), and the diagonals bisect each other. **Converses:** opposite angles equal, OR opposite sides equal, OR diagonals bisecting each other → it IS a parallelogram.

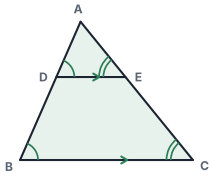
3. Circles

[BC] a diameter ⇒ ∠BAC = 90° (Corollary 3), and the converse (Corollary 4). **Higher Level:** angle at the centre = 2 × angle at the circle on the same arc (Theorem 19); angles on the same arc are equal (Corollary 2); opposite angles of a cyclic quadrilateral add to 180° (Corollary 5).

4. Congruent triangles (same shape AND size)

SSS three sides · **SAS** two sides and the angle **between** them · **ASA** two angles and the side **between** them · **RHS** right angle, hypotenuse, one other side. **Not tests:** AAA, and SSA (angle not between the sides). △ABC ≡ △PQR: A↔P, B↔Q, C↔R.

5. Similar triangles (same shape)



Equal angles ⇒ similar ⇒ sides in proportion (Theorem 13). **Scale factor** = new side ÷ matching old side. DE ∥ BC makes △ADE similar to △ABC. **Higher Level:** DE ∥ BC cuts [AB] and [AC] in the same ratio (Theorem 12).

Classic mistakes: adding instead of multiplying for similar triangles · using |AD| instead of the whole |AB| · SAS with the angle not between the sides · matching the letters in the wrong order · 180° for the angle in a semicircle.

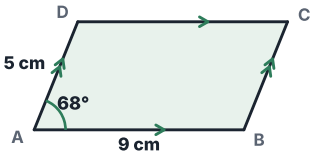
Exam tip: one fact per line with a **reason** in brackets. In a congruence proof list three facts, then name the test: "△ABD ≡ △ACD (SSS)".

PRACTICE · 34 QUESTIONS

Have a go

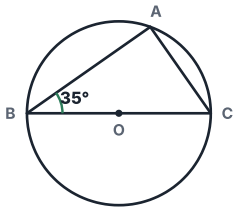
↗ Léim

A. Parallelograms



1. Use the diagram (ABCD is a parallelogram). Find ∠BCD. _____
2. Find ∠ABC. _____
3. Find |DC|. _____
4. Find the perimeter of ABCD. _____
5. The diagonals of ABCD meet at O and |AO| = 6.5 cm. Find |AC|. _____
6. |BD| = 8.4 cm. Find |OD|. _____
7. PQRS is a parallelogram with ∠P = (2x + 10)° and ∠R = 74°. Find x. _____
8. In a parallelogram, one angle is 3 times the angle next to it. Find the smaller angle. _____

B. Circles

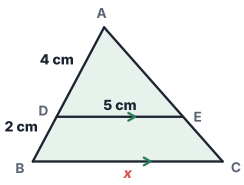


9. Use the diagram ([BC] is a diameter). Find ∠BAC. _____
10. Find ∠ACB. _____
11. A, B and C are on a circle and ∠BAC = 90°. What can you say about [BC]? _____
12. [BC] is a diameter, ∠ABC = 3x and ∠ACB = 2x. Find x. _____
13. *Higher Level:* O is the centre and ∠AOB = 140°. C is on the major arc. Find ∠ACB. _____
14. *Higher Level:* C and D are on the circle, on the same side of [AB], and ∠ADB = 47°. Find ∠ACB. _____
15. *Higher Level:* ABCD is a cyclic quadrilateral with ∠DAB = 104°. Find ∠BCD. _____

C. Congruent triangles

16. Which test proves △ABC ≡ △PQR, if any? |AB| = |PQ| = 5 cm, |BC| = |QR| = 7 cm, |AC| = |PR| = 9 cm. _____
17. Which test proves △ABC ≡ △PQR, if any? |AB| = |PQ| = 6 cm, ∠ABC = ∠PQR = 50°, |BC| = |QR| = 8 cm. _____
18. Which test proves △ABC ≡ △PQR, if any? ∠A = ∠P = 40°, ∠B = ∠Q = 75°, |AB| = |PQ| = 10 cm. _____
19. Which test proves △ABC ≡ △PQR, if any? ∠A = ∠P = 90°, |BC| = |QR| = 13 cm, |AB| = |PQ| = 5 cm. _____
20. Which test proves △ABC ≡ △PQR, if any? ∠A = ∠P = 50°, ∠B = ∠Q = 60°, ∠C = ∠R = 70°. _____
21. △ABC ≡ △XYZ. Which side is equal to [BC]? _____
22. △ABC ≡ △XYZ and ∠ABC = 64°. Find ∠XYZ. _____
23. What is the hypotenuse of a right-angled triangle? _____

D. Similar triangles



24. Use the diagram (DE ∥ BC). Find x = |BC|. _____
25. In the diagram, |AE| = 3 cm. Find |AC|. _____
26. A triangle has sides 4 cm, 6 cm and 7 cm. In a similar triangle the shortest side is 10 cm. Find the longest side. _____
27. Are triangles with sides 3, 5, 6 and 6, 10, 12 similar? _____
28. One triangle has angles 50° and 60°; another has angles 60° and 70°. Are they similar? _____
29. Seán is 1.8 m tall and his shadow is 2.4 m long. At the same time a building's shadow is 16 m long. How tall is the building? _____
30. *Higher Level:* In △ABC, DE ∥ BC and |AD| : |DB| = 3 : 2. |AC| = 15 cm. Find |AE|. _____

Questions 31–34 are on the next page.

EXAM-STYLE · THEN CHECK

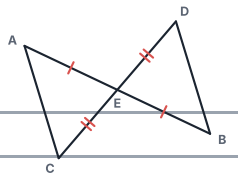
Exam corner + answers

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E. Exam-style questions (show your work, with reasons)

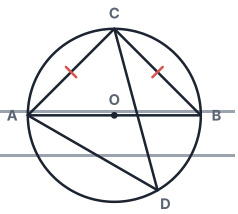
31. **A parallelogram.** ABCD is a parallelogram with ∠ABC = 72°. Its diagonals meet at M. |AM| = 3.5 cm and |BD| = 11 cm. Find (a) ∠BCD (b) ∠ADC (c) |AC| (d) |BM|. Give a reason for each.

32. **A proof.** [AB] and [CD] bisect each other at E (see the diagram). (a) Prove that △AEC ≡ △BED. (b) Hence show that AC ∥ DB.



33. **Similar triangles.** In △PQR, S is on [PQ], T is on [PR] and ST ∥ QR. |PS| = 8 cm, |SQ| = 4 cm and |ST| = 10 cm. (a) Find |QR|. (b) |PR| = 18 cm. Find |PT|.

34. **A circle.** [AB] is a diameter, C is on the circle and |AC| = |BC| (diagram). (a) Find ∠ACB. (b) Find ∠CAB. (c) *Higher Level:* D is on the circle. Find ∠ADC.



Answers

A	1. 68° · 2. 112° · 3. 9 cm · 4. 28 cm · 5. 13 cm · 6. 4.2 cm · 7. 32 · 8. 45°
B	9. 90° · 10. 55° · 11. it's a diameter (Corollary 4) · 12. 18 · 13. 70° · 14. 47° · 15. 76°
C	16. SSS · 17. SAS · 18. ASA · 19. RHS · 20. not enough (AAA) · 21. [YZ] · 22. 64° · 23. the side opposite the right angle
D	24. 7.5 cm · 25. 4.5 cm · 26. 17.5 cm · 27. yes: every side × 2 · 28. yes: both are 50°, 60°, 70° · 29. 12 m · 30. 9 cm
E	31. (a) **108°** (co-interior) (b) **72°** (opposite angles) (c) **7 cm** (d) **5.5 cm** (diagonals bisect each other) · 32. (a) |AE| = |EB| and |CE| = |ED| (E is the midpoint of both); ∠AEC = ∠BED (vertically opposite) → **SAS**. (b) ∠CAE = ∠DBE (matching angles), and these are alternate angles, so **AC ∥ DB**. · 33. (a) scale factor 12 ÷ 8 = 1.5: **15 cm** (b) **12 cm** · 34. (a) **90°** (angle in a semicircle) (b) (180° − 90°) ÷ 2 = **45°** (isosceles) (c) **45°** (same arc AC as ∠ABC = 45°)

Got one wrong? Similar triangles: did you multiply by the scale factor, and use the WHOLE side? Congruence: is the angle between the two sides? Then play **Proof Builder** at leim.ie/proof-builder.html: the "Go back over these" list at the end shows the reason for each mistake.

All questions written by Léim. Léim is independent and not connected with the State Examinations Commission or the NCCA.